Properties : **(i)** $L[u(t-a)] = \frac{e^{-as}}{s}$

(ii) $L[f(t-a)u(t-a)] = e^{-as}\bar{f}(s)$ where $L[f(t)] = \bar{f}(s)$

(iii) If $f(t) = \begin{cases} f_1(t), & t \le a \\ f_2(t), & t > a \end{cases}$

Then $f(t) = f_1(t) + [f_2(t) - f_1(t)]u(t-a)$

(iv) If $f(t) = \begin{cases} f_1(t), & t \le a \\ f_2(t), & a < t \le b \\ f_3(t), & t > b \end{cases}$

Then $f(t) = f_1(t) + [f_2(t) - f_1(t)]u(t-a) + [f_3(t) - f_2(t)]u(t-b)$

Unit impulse function (Dirac delta function)

➢ $\delta(t-a) = \lim_{\varepsilon \to 0} \delta_\varepsilon(t-a)\ ;\ a \ge 0$

where $\delta_\varepsilon(t-a) = \begin{cases} \frac{1}{\varepsilon} & \text{if } a \le t \le a+\varepsilon \\ 0 & \text{otherwise} \end{cases}$

➢ We have $L[\delta(t-a)] = e^{-as}$

Unit - VIII : Laplace Transforms - 2

Table of inverse Laplace transforms

	Function	Inverse Transform		Function	Inverse Transform
1	$\frac{1}{s}$	1	5	$\frac{1}{s^2+a^2}$	$\frac{\sin at}{a}$
2	$\frac{1}{s-a}$	e^{at}	6	$\frac{1}{s^2-a^2}$	$\frac{\sinh at}{a}$
3	$\frac{s}{s^2+a^2}$	$\cos at$	7	$\frac{1}{s^{n+1}}\ (n > -1)$	$\frac{t^n}{\Gamma(n+1)}$
4	$\frac{s}{s^2-a^2}$	$\cosh at$	8	$\frac{1}{s^{n+1}}\ (n = 1, 2, 3 \ldots)$,	$\frac{t^n}{n!}$

Properties of inverse Laplace transforms

(i) $L^{-1}[e^{-as}\bar{f}(s)] = f(t-a)\,u(t-a)$

(ii) $L^{-1}[\bar{f}(s-a)] = e^{at}L^{-1}[\bar{f}(s)]$

(iii) $L^{-1}[-\bar{f}'(s)] = t\,f(t)$; $L^{-1}\left[\bar{f}''(s)\right] = t^2 f(t)$

(iv) $L^{-1}\left[\dfrac{\bar{f}(s)}{s}\right] = \displaystyle\int_0^t f(t)\,dt$

Convolution of two functions $f(t)$ and $g(t)$

$$f(t) * g(t) = \int_{u=0}^{t} f(u)\,g(t-u)\,du$$

Property : $f(t) * g(t) = g(t) * f(t)$

Convolution theorem

If $L^{-1}[\bar{f}(s)] = f(t)$ and $L^{-1}[\bar{g}(s)] = g(t)$ then

$$L^{-1}[\bar{f}(s)\cdot\bar{g}(s)] = \int_{u=0}^{t} f(u)\,g(t-u)\,du = f(t) * g(t)$$

or

$$L\left[\int_0^t f(u)\,g(t-u)\,du\right] = \bar{f}(s)\cdot\bar{g}(s) = L\left[\int_0^t f(t-u)\,g(u)\,du\right]$$

Laplace transform of the derivatives

(i) $L[y'(t)] = s\,L\,y(t) - y(0)$

(ii) $L[y''(t)] = s^2 L\,y(t) - sy(0) - y'(0)$

(iii) $L[y'''(t)] = s^3 L\,y(t) - s^2 y(0) - s\,y'(0) - y''(0)$ etc.

These are used for solving initial value problems using Laplace transforms.